

Modified Holographic Dark Energy

Hao Wei*

Department of Physics, Beijing Institute of Technology, Beijing 100081, China

ABSTRACT

In this work, motivated by the energy bound suggested by Cohen *et al.*, we propose the modified holographic dark energy (MHDE) model. Choosing the IR cut-off $L = R_{\text{CC}}$ and considering the parameterizations $n^2 = 2 - \lambda a$, $n^2 = 2 - 3\lambda a^2 / (1 + 3a^2)$ and $n^2 = 2 - \lambda a^2 / (\beta + a^2)$, we derive all the physical quantities of the non-saturated MHDE model analytically. We find that the non-saturated MHDE models with the parameterizations $n^2 = 2 - \lambda a$ and $n^2 = 2 - 3\lambda a^2 / (1 + 3a^2)$ are single-parameter models in practice. Also, we consider the cosmological constraints on the non-saturated MHDE, and find that it is well consistent with the observational data.

PACS numbers: 95.36.+x, 98.80.Es, 98.80.-k

* email address: haowei@bit.edu.cn

I. INTRODUCTION

Recently, the holographic dark energy (HDE) has been considered as an interesting candidate of dark energy, which has been studied extensively in the literature. It was proposed from the well-known holographic principle [1, 2] in the string theory. For a quantum gravity system, the local quantum field cannot contain too many degrees of freedom, otherwise the formation of black hole is inevitable and then the quantum field theory breaks down. In the black hole thermodynamics [3, 4], there is a maximum entropy in a box of size L , namely the Bekenstein-Hawking entropy bound S_{BH} , which scales as the area of the box $\sim L^2$, rather than the volume $\sim L^3$. To avoid the breakdown of the local quantum field theory, Cohen *et al.* [5] proposed a more restrictive bound, namely the energy bound. If ρ_Λ is the quantum zero-point energy density caused by a short distance cut-off, the total energy in a box of size L cannot exceed the mass of a black hole of the same size [5], namely $\rho_\Lambda L^3 \lesssim m_p^2 L$, where $m_p \equiv (8\pi G)^{-1/2}$ is the reduced Planck mass. The largest IR cut-off L is the one saturating the inequality. Thus,

$$\rho_\Lambda = 3n^2 m_p^2 L^{-2}, \quad (1)$$

where the numerical constant $3n^2$ is introduced for convenience. If we choose L as the size of the universe, for instance the Hubble horizon H^{-1} , the resulting ρ_Λ is comparable to the observational density of dark energy [6, 7]. However, Hsu [7] pointed out that in this case the resulting equation-of-state parameter (EoS) is equal to zero, which cannot accelerate the expansion of the universe. The other possibility [8] is to choose L as the particle horizon

$$R_H \equiv a \int_0^t \frac{d\tilde{t}}{a} = a \int_0^a \frac{d\tilde{a}}{H\tilde{a}^2}, \quad (2)$$

where $H \equiv \dot{a}/a$ is the Hubble parameter; $a = (1+z)^{-1}$ is the scale factor of the universe; z is the redshift; we have set $a_0 = 1$; the subscript “0” indicates the present value of the corresponding quantity; a dot denotes the derivative with respect to cosmic time t . However, it is easy to find that the EoS is always larger than $-1/3$ and also cannot accelerate the expansion of the universe in this case with $L = R_H$ [9]. To get an accelerating universe, Li proposed in [9] to choose L as the future event horizon

$$R_h \equiv a \int_t^\infty \frac{d\tilde{t}}{a} = a \int_a^\infty \frac{d\tilde{a}}{H\tilde{a}^2}. \quad (3)$$

In this case, the EoS of the holographic dark energy can be less than $-1/3$ [9]. In fact, the version of HDE with $L = R_h$ is the one considered extensively in the literature. For a comprehensive list of references concerning HDE, we refer to e.g. [10, 11, 12, 36, 38] and references therein.

In the literature, many authors extensively considered the HDE whose energy density was taken to be the one given in Eq. (1). Here, we instead propose to consider a modified version of HDE. Notice that the energy bound proposed in [5] requires that the *total* energy in a box of size L cannot exceed the mass of a black hole of the same size. Therefore, it is reasonable to argue that when HDE coexists with matter, the total energy should satisfy the energy bound, namely, $\rho_{tot} L^3 \lesssim m_p^2 L$. Correspondingly, the largest IR cut-off L is the one saturating the inequality. Thus,

$$\rho_{tot} = 3n^2 m_p^2 L^{-2}, \quad (4)$$

where $\rho_{tot} = \rho_m + \rho_\Lambda$ is the total energy density; $\rho_m = \rho_{m0} a^{-3}$ is the energy density of matter. Therefore, the energy density of the modified HDE is given by $\rho_\Lambda = \rho_{tot} - \rho_m = 3n^2 m_p^2 L^{-2} - \rho_{m0} a^{-3}$ instead. Note that we do not consider the radiation in this work, since almost all the cosmological observations probe the cosmic history after the matter-radiation equality. Using the Friedmann equation $3m_p^2 H^2 = \rho_{tot}$ and Eq. (4), it is easy to find that $H^2 L^2 = n^2$. Note that we consider a flat universe throughout. Since n is given in the form of n^2 throughout, we only consider the positive n in this work. So, we have

$$HL = n. \quad (5)$$

This very simple equation is our starting point for the modified HDE (hereafter MHDE). Naturally, the next step is to choose an appropriate L .

II. CHOOSING AN APPROPRIATE IR CUT-OFF

The most natural choice for L is $L = H^{-1}$. In this case, one should have $n = 1$ from Eq. (5). So, the requirement of the holographic principle, namely Eq. (4), is nothing but the familiar Friedmann equation. There is no new input. Therefore, the choice $L = H^{-1}$ is trivial.

The second choice for L is the particle horizon R_H in Eq. (2). For $L = R_H$, we have

$$\frac{dL}{da} = \frac{L}{a} + \frac{1}{Ha} = \frac{L}{a} \left(1 + \frac{1}{n}\right), \quad (6)$$

which can be regarded as a differential equation for L with respect to a . From Eq. (6), one can find that $L \propto a^{1+n^{-1}}$ and $H = n/L \propto a^{-1-n^{-1}}$. Noting that in the matter-dominated epoch $H \propto a^{-3/2}$, MHDE with $L = R_H$ can describe the matter-dominated epoch when $n = 2$. This is a good news. However, we should consider the second thing. As is well known,

$$\frac{\ddot{a}}{a} = \dot{H} + H^2 = H^2 + Ha \frac{dH}{da}. \quad (7)$$

Substituting $H \propto a^{-1-n^{-1}}$ into Eq. (7), we have $\ddot{a}/a = -n^{-1}H^2 < 0$. Therefore, the universe cannot be accelerated, and hence the choice $L = R_H$ is unsuitable.

The third choice for L is the future event horizon R_h in Eq. (3). For $L = R_h$, we have

$$\frac{dL}{da} = \frac{L}{a} - \frac{1}{Ha} = \frac{L}{a} \left(1 - \frac{1}{n}\right). \quad (8)$$

So, it is easy to see that $L \propto a^{1-n^{-1}}$ and $H = n/L \propto a^{-1+n^{-1}}$. Unfortunately, it cannot describe the matter-dominated epoch in which $H \propto a^{-3/2}$. On the other hand, substituting $H \propto a^{-1+n^{-1}}$ into Eq. (7), we have $\ddot{a}/a = n^{-1}H^2 > 0$. Thus, the universe is accelerating, but cannot undergo a decelerated epoch. So, the choice $L = R_h$ is also unsuitable.

There is another choice. In [13], Gao *et al.* suggested that L can be proportional to the Ricci scalar curvature radius. The resulting HDE was called Ricci dark energy in the literature. See e.g. [14, 15, 16, 17] for works on the so-called Ricci dark energy. In [13, 14, 15, 16, 17], there is no physical motivation to this proposal for L in fact. Recently, in [18], Cai *et al.* found that the Jeans length R_{CC} which is determined by $R_{CC}^{-2} = \dot{H} + 2H^2$ gives the causal connection scale of perturbations in the flat universe. Since the Ricci scalar is also proportional to $\dot{H} + 2H^2$ in the flat universe, Cai *et al.* in fact found the physical motivation for the Ricci dark energy. Here, we take $L = R_{CC}$ as the fourth choice for MHDE. In this case, from Eq. (5), we obtain

$$H^2 = n^2 L^{-2} = n^2 \left(\dot{H} + 2H^2 \right). \quad (9)$$

It can be recast as a differential equation for H with respect to a , namely

$$\frac{dH}{da} = \left(\frac{1}{n^2} - 2 \right) \frac{H}{a}. \quad (10)$$

So, we find that $H \propto a^{n^{-2}-2}$. When $n^2 = 2$, MHDE with $L = R_{CC}$ can describe the matter-dominated epoch in which $H \propto a^{-3/2}$. On the other hand, substituting $H \propto a^{n^{-2}-2}$ into Eq. (7), we find that

$$\frac{\ddot{a}}{a} = \left(\frac{1}{n^2} - 1 \right) H^2. \quad (11)$$

Thus, the universe is decelerating when $n > 1$, whereas the universe is accelerating when $n < 1$. So, the choice $L = R_{CC}$ gives the possibility for a cosmologically feasible MHDE model.

However, in the saturated MHDE model, n is constant. For a given n , it is impossible to describe both the decelerated and accelerated phases, even in the case with $L = R_{CC}$. To build a cosmologically feasible MHDE model, we should go further.

III. NON-SATURATED MHDE

In the derivation of HDE mentioned in Sec. I, the holographic bound was chosen to be saturated. However, this is not necessary. In [19], Guberina *et al.* proposed the so-called non-saturated HDE, in which the parameter $n = n(t)$ is a function of cosmic time t , rather than a constant. Similarly, here we consider the non-saturated MHDE with $L = R_{CC}$, in which $n = n(a)$ is a function of scale factor a . To solve Eq. (10), we should specify the explicit form of $n(a)$. Similar to the well-known EoS parameterization $w = w_0 + w_a(1-a) = w_1 - w_2 a$ considered extensively in the literature [20], we parameterize $n^2 = \alpha - \lambda a$, where α and λ are constants. In fact, one can regard this type of parameterization as a linearized expansion with respect to scale factor a . As shown in Sec. II, when $n^2 = 2$, we find that MHDE with $L = R_{CC}$ can describe the matter-dominated epoch in which $H \propto a^{-3/2}$. So, we get $\alpha = 2$ by requiring $n^2 \rightarrow 2$ when $a \rightarrow 0$. From now on, we consider the non-saturated MHDE in which $L = R_{CC}$ and

$$n^2 = 2 - \lambda a. \quad (12)$$

Noting that $n^2 > 0$, we should require $\lambda < 2$ at least to describe the cosmic history up to now, namely $0 \leq z < \infty$. Substituting Eq. (12) into Eq. (10), we can recast it as

$$\frac{dH}{da} = \left(\frac{1}{2 - \lambda a} - 2 \right) \frac{H}{a}. \quad (13)$$

The solution reads

$$H = H_0 (2 - \lambda)^{1/2} a^{-3/2} (2 - \lambda a)^{-1/2}, \quad (14)$$

where $H_0 = H(z=0)$ is the Hubble constant. Obviously, $H \propto a^{-3/2}$ when $a \rightarrow 0$, and hence this model can describe the matter-dominated epoch. For convenience, we rewrite Eq. (14) to

$$E^2 = (2 - \lambda) (1 + z)^3 \left(2 - \frac{\lambda}{1 + z} \right)^{-1}, \quad (15)$$

where $E \equiv H/H_0$. Substituting Eq. (14) into Eq. (7), we can obtain the deceleration parameter

$$q \equiv -\frac{\ddot{a}}{aH^2} = \frac{1}{2} \left(1 - \frac{\lambda a}{2 - \lambda a} \right). \quad (16)$$

Obviously, the universe is decelerating when $\lambda a < 1$, whereas the universe is accelerating when $\lambda a > 1$. The transition occurs when $\lambda a = 1$. Therefore, the transition redshift is given by

$$z_t = \lambda - 1. \quad (17)$$

From the Friedmann equation $3m_p^2 H^2 = \rho_{tot} = \rho_m + \rho_\Lambda$, we have $\rho_\Lambda = 3m_p^2 H^2 - \rho_{m0} a^{-3}$, or equivalently

$$\tilde{\rho}_\Lambda \equiv \frac{\rho_\Lambda}{3m_p^2 H_0^2} = (2 - \lambda) (1 + z)^3 \left(2 - \frac{\lambda}{1 + z} \right)^{-1} - \Omega_{m0} (1 + z)^3. \quad (18)$$

The fractional energy density $\Omega_i \equiv \rho_i / (3m_p^2 H^2)$, where $i = m$ and Λ . In particular, we find that

$$\Omega_m = \frac{\Omega_{m0} a^{-3}}{E^2} = \frac{\Omega_{m0}}{2 - \lambda} \left(2 - \frac{\lambda}{1 + z} \right). \quad (19)$$

Requiring $\Omega_m \rightarrow 1.0$ (namely matter dominated) when $z \rightarrow \infty$, we have

$$\Omega_{m0} = 1 - \frac{\lambda}{2}. \quad (20)$$

Unlike the other versions of HDE considered in the literature, Ω_{m0} is *not* an independent parameter in the non-saturated MHDE model. This is an important result. Using Eq. (20), we finally obtain

$$\Omega_m = 1 - \frac{\lambda}{2(1 + z)}, \quad (21)$$

$$\Omega_\Lambda = 1 - \Omega_m = \frac{\lambda}{2(1 + z)}. \quad (22)$$

Interestingly, we find that $\Omega_\Lambda = \Omega_m$ at the transition redshift z_t given in Eq. (17) which is determined by $q = 0$. Provided that λ is of order unity, the cosmological coincidence problem could be alleviated. On the other hand, it is easy to find the effective EoS as (see e.g. [21, 37])

$$w_{\text{eff}} \equiv \frac{p_{\text{tot}}}{\rho_{\text{tot}}} = -1 - \frac{2}{3} \frac{\dot{H}}{H^2} = \frac{1}{3} (2q - 1) = -\frac{\lambda a}{3(2 - \lambda a)}, \quad (23)$$

Noting that $w_{\text{eff}} = \Omega_m w_m + \Omega_\Lambda w_\Lambda$ and $w_m = 0$, the EoS of HDE reads

$$w_\Lambda = \frac{w_{\text{eff}}}{\Omega_\Lambda}, \quad (24)$$

where Ω_Λ is given in Eq. (22). If λ is given, all the physical quantities can be derived accordingly, thanks to the important result Eq. (20). In fact, to our knowledge, the non-saturated MHDE model considered in this work is the fourth *single-parameter* cosmological model besides the well-known flat Λ CDM model, the flat DGP braneworld model [22] and the new agegraphic dark energy (NADE) model [23].

IV. COSMOLOGICAL CONSTRAINTS ON THE NON-SATURATED MHDE

Here, we consider the cosmological constraints on the non-saturated MHDE. In [24, 25], Supernova Cosmology Project (SCP) collaboration released their new dataset of type Ia supernovae (SNIa), which was called Union compilation. The Union compilation contains 414 SNIa and reduces to 307 SNIa after selection cuts [24].

We perform a χ^2 analysis to obtain the constraints on the single parameter λ of the non-saturated MHDE model. The data points of the 307 Union SNIa compiled in [24] are given in terms of the distance modulus $\mu_{\text{obs}}(z_i)$. On the other hand, the theoretical distance modulus is defined as

$$\mu_{\text{th}}(z_i) \equiv 5 \log_{10} D_L(z_i) + \mu_0, \quad (25)$$

where $\mu_0 \equiv 42.38 - 5 \log_{10} h$ and h is the Hubble constant H_0 in units of 100 km/s/Mpc, whereas

$$D_L(z) = (1+z) \int_0^z \frac{d\tilde{z}}{E(\tilde{z}; \lambda)}, \quad (26)$$

The χ^2 from the 307 Union SNIa are given by

$$\chi_\mu^2(\lambda) = \sum_i \frac{[\mu_{\text{obs}}(z_i) - \mu_{\text{th}}(z_i)]^2}{\sigma^2(z_i)}, \quad (27)$$

where σ is the corresponding 1σ error. The parameter μ_0 is a nuisance parameter but it is independent of the data points. One can perform an uniform marginalization over μ_0 . However, there is an alternative way. Following [26, 27], the minimization with respect to μ_0 can be made by expanding the χ_μ^2 of Eq. (27) with respect to μ_0 as

$$\chi_\mu^2(\lambda) = \tilde{A} - 2\mu_0 \tilde{B} + \mu_0^2 \tilde{C}, \quad (28)$$

where

$$\begin{aligned} \tilde{A}(\lambda) &= \sum_i \frac{[\mu_{\text{obs}}(z_i) - \mu_{\text{th}}(z_i; \mu_0 = 0, \lambda)]^2}{\sigma_{\mu_{\text{obs}}}^2(z_i)}, \\ \tilde{B}(\lambda) &= \sum_i \frac{\mu_{\text{obs}}(z_i) - \mu_{\text{th}}(z_i; \mu_0 = 0, \lambda)}{\sigma_{\mu_{\text{obs}}}^2(z_i)}, \quad \tilde{C} = \sum_i \frac{1}{\sigma_{\mu_{\text{obs}}}^2(z_i)}. \end{aligned}$$

Eq. (28) has a minimum for $\mu_0 = \tilde{B}/\tilde{C}$ at

$$\tilde{\chi}_\mu^2(\lambda) = \tilde{A}(\lambda) - \frac{\tilde{B}(\lambda)^2}{\tilde{C}}. \quad (29)$$

Since $\chi_{\mu, \min}^2 = \tilde{\chi}_{\mu, \min}^2$ obviously, we can instead minimize $\tilde{\chi}_\mu^2$ which is independent of μ_0 . Note that the above summations are over the 307 Union SNIa compiled in [24]. It is worth noting that the corresponding h can be determined by $\mu_0 = \tilde{B}/\tilde{C}$ for the best-fit parameter.

At first, we consider the cosmological constraints on the non-saturated MHDE, by using the 307 Union SNIa dataset only. In the top-left panel of Fig. 1, we present the likelihood $\mathcal{L} \propto e^{-\chi^2/2}$ versus λ . We find that the best-fit model parameter is $\lambda = 1.502_{-0.030}^{+0.028}$ (with 1σ uncertainty) or $\lambda = 1.502_{-0.063}^{+0.054}$ (with 2σ uncertainty), while $\chi_{\min}^2 = 311.272$. The corresponding $h = 0.707$. We plot $H(z)$ for the best-fit parameter λ in the top-right panel of Fig. 1. We also present the observational $H(z)$ data from [28] (see also [21, 29, 30]) for comparison. It is easy to see that $H(z)$ of the non-saturated MHDE is well consistent with the observational $H(z)$ data. In addition, Ω_m , Ω_Λ , q , w_{eff} and w_Λ as functions of redshift z for the best-fit λ are also presented in Fig. 1. Clearly, Ω_Λ and Ω_m are comparable since the near past, whereas $\Omega_{m0} = 1 - \lambda/2 = 0.249$ and $\Omega_{\Lambda0} = \lambda/2 = 0.751$. We see that q changed from $q > 0$ to $q < 0$ at $z_t = \lambda - 1 = 0.502$. On the other hand, w_Λ crossed the so-called phantom divide $w = -1$.

In the literature, the shift parameter R from the cosmic microwave background (CMB) anisotropy, and the distance parameter A of the measurement of the baryon acoustic oscillation (BAO) peak in the distribution of SDSS luminous red galaxies [31], are also used extensively in obtaining the cosmological constraints. The shift parameter R is defined by [32, 33]

$$R \equiv \Omega_{m0}^{1/2} \int_0^{z_*} \frac{d\tilde{z}}{E(\tilde{z})}, \quad (30)$$

where the redshift of recombination $z_* = 1090$ which has been updated in the 5-year data of Wilkinson Microwave Anisotropy Probe (WMAP5) [34]. The shift parameter R relates the angular diameter distance to the last scattering surface, the comoving size of the sound horizon at z_* and the angular scale of the first acoustic peak in the CMB power spectrum of the temperature fluctuations [32, 33]. The value of R has been updated to 1.710 ± 0.019 in WMAP5 [34]. The distance parameter A is given by

$$A \equiv \Omega_{m0}^{1/2} E(z_b)^{-1/3} \left[\frac{1}{z_b} \int_0^{z_b} \frac{d\tilde{z}}{E(\tilde{z})} \right]^{2/3}, \quad (31)$$

where $z_b = 0.35$. In [35], the value of A has been determined to be $0.469 (n_s/0.98)^{-0.35} \pm 0.017$. Here the scalar spectral index n_s is taken to be 0.960 which has been updated in WMAP5 [34].

Now, we perform a joint χ^2 analysis to obtain the cosmological constraints on the single parameter λ of the non-saturated MHDE model, by using the combined data of the 307 Union SNIa, the shift parameter R of CMB and the distance parameter A of BAO. Here, the total χ^2 is given by

$$\chi^2 = \tilde{\chi}_\mu^2 + \chi_{CMB}^2 + \chi_{BAO}^2, \quad (32)$$

where $\tilde{\chi}_\mu^2$ is given in Eq. (29), $\chi_{CMB}^2 = (R - R_{\text{obs}})^2/\sigma_R^2$ and $\chi_{BAO}^2 = (A - A_{\text{obs}})^2/\sigma_A^2$. The best-fit model parameter can be determined by minimizing the total χ^2 . In the top-left panel of Fig. 2, we present the likelihood $\mathcal{L} \propto e^{-\chi^2/2}$ versus λ . We find that the best-fit model parameter is $\lambda = 1.461_{-0.029}^{+0.028}$ (with 1σ uncertainty) or $\lambda = 1.461_{-0.061}^{+0.054}$ (with 2σ uncertainty), while $\chi_{\min}^2 = 320.858$. The corresponding $h = 0.699$. We also plot $H(z)$ for the best-fit λ in the top-right panel of Fig. 2. Again, it is easy to see that $H(z)$ of the non-saturated MHDE is well consistent with the observational $H(z)$ data. In addition, Ω_m , Ω_Λ , q , w_{eff} and w_Λ as functions of redshift z for the best-fit λ are also presented in Fig. 2. Clearly, Ω_Λ and Ω_m are comparable since the near past, whereas $\Omega_{m0} = 1 - \lambda/2 = 0.269$ and $\Omega_{\Lambda0} = \lambda/2 = 0.731$. We see that q changed from $q > 0$ to $q < 0$ at $z_t = \lambda - 1 = 0.461$. On the other hand, w_Λ crossed the so-called phantom divide $w = -1$.

As shown above, the non-saturated MHDE with the simplest parameterization $n^2 = 2 - \lambda\alpha$ is well consistent with the observational data in fact. This is fairly impressive, since all the physical quantities of this model are given by the simple and analytical expressions.

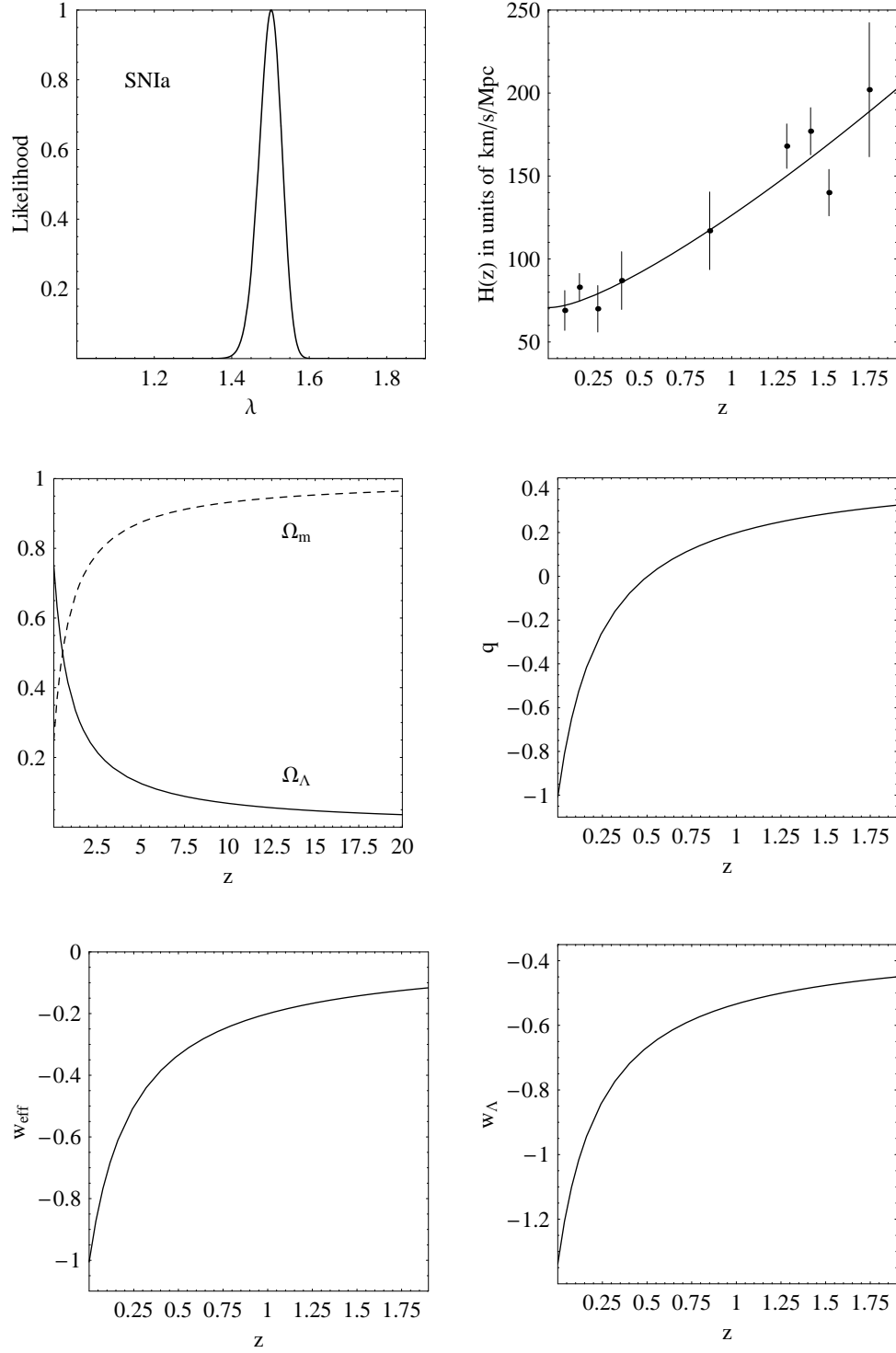


FIG. 1: The top-left panel is the likelihood $\mathcal{L} \propto e^{-\chi^2/2}$ versus λ . The other panels are H , Ω_m , Ω_Λ , q , w_{eff} and w_Λ as functions of redshift z for the best-fit λ . These results are obtained by using the SNIa data only.

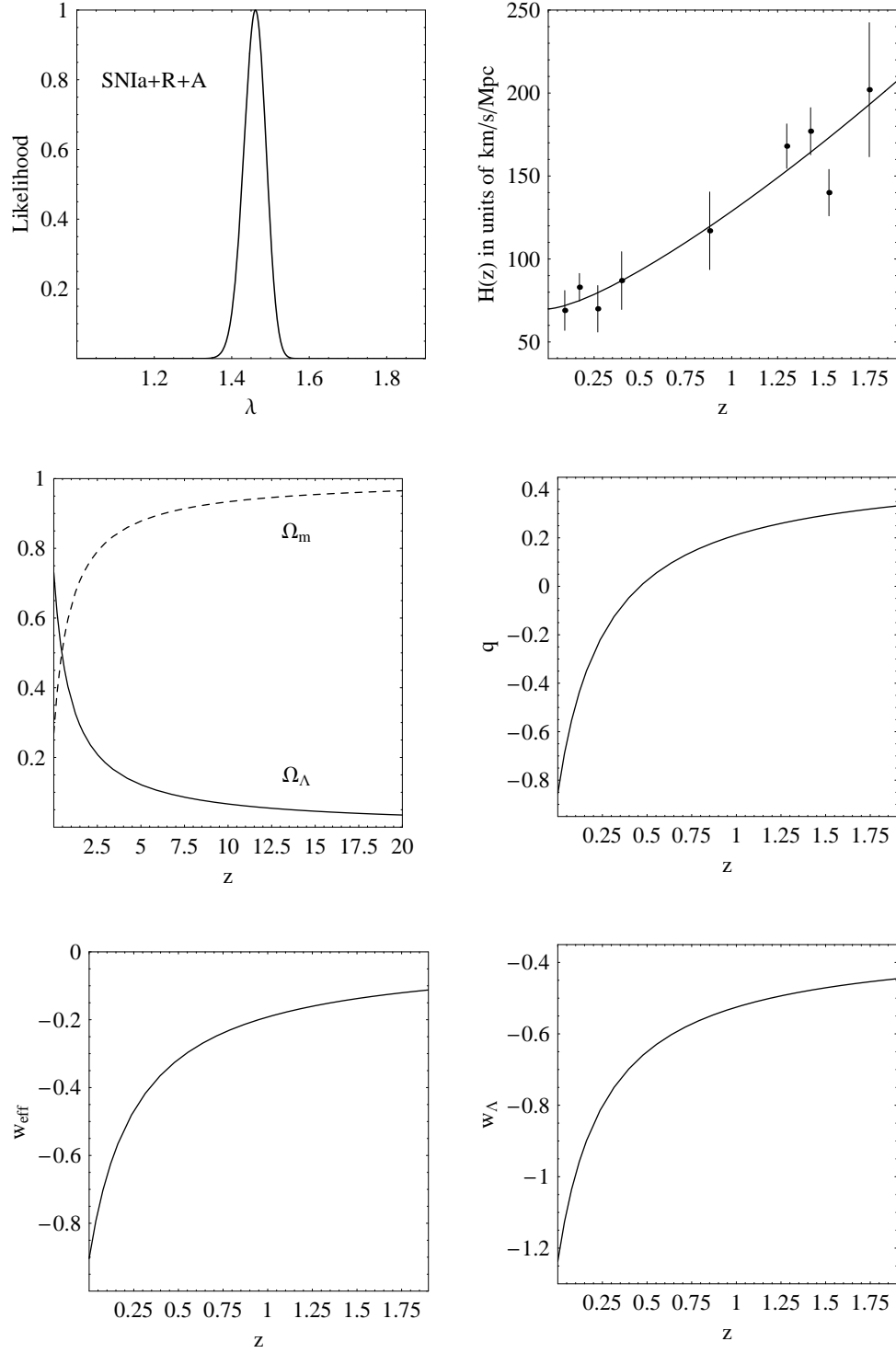


FIG. 2: The same as in Fig. 1, except that these results are obtained by using the combined data of the 307 Union SNIa, the shift parameter R of CMB and the distance parameter A of BAO.

V. OTHER PARAMETERIZATIONS FOR THE NON-SATURATED MHDE

Although the simplest parameterization $n^2 = 2 - \lambda a$ given in Eq. (12) has been shown that it is fairly successful to fit the cosmological observations, it is only valid for $a < 2/\lambda$ which is required by $n^2 > 0$. So, we cannot use it to describe the future evolution of the universe. In fact, it is non-trivial to find a suitable parameterization of n which is well-defined for the whole $0 \leq a < \infty$. Naively, one can consider the simplest parameterization $n^2 = 2 - \lambda a / (\beta + a)$. However, we find that it cannot be consistent with the combined observational data of the 307 Union SNIa, the shift parameter R of CMB and the distance parameter A of BAO. Here, we consider another parameterization $n^2 = \alpha - \lambda a^2 / (\beta + a^2)$, where α , β and λ are constants. Again, we obtain $\alpha = 2$ by requiring $n^2 \rightarrow 2$ when $a \rightarrow 0$, in order to describe the matter-dominated epoch in which $H \propto a^{-3/2}$. From now on, we consider the non-saturated MHDE in which $L = R_{CC}$ and

$$n^2 = 2 - \frac{\lambda a^2}{\beta + a^2}. \quad (33)$$

We require $\beta \geq 0$ to avoid singularity. Noting that $0 \leq a^2 / (\beta + a^2) < 1$ for $0 \leq a < \infty$, we also require $\lambda < 2$ to ensure $n^2 > 0$. Substituting Eq. (33) into Eq. (10), we find that the solution is given by

$$H = H_0 a^{-3/2} \left[\frac{2\beta + (2 - \lambda) a^2}{2\beta + 2 - \lambda} \right]^{\frac{\lambda}{4(2 - \lambda)}}. \quad (34)$$

Obviously, $H \propto a^{-3/2}$ when $a \rightarrow 0$, and hence this model can describe the matter-dominated epoch. Then, we have

$$E^2 = (1 + z)^3 \left[\frac{2\beta + (2 - \lambda) (1 + z)^{-2}}{2\beta + 2 - \lambda} \right]^{\frac{\lambda}{2(2 - \lambda)}}. \quad (35)$$

On the other hand,

$$\Omega_m = \frac{\Omega_{m0} a^{-3}}{E^2} = \Omega_{m0} \left[\frac{2\beta + 2 - \lambda}{2\beta + (2 - \lambda) (1 + z)^{-2}} \right]^{\frac{\lambda}{2(2 - \lambda)}}. \quad (36)$$

Again, requiring $\Omega_m \rightarrow 1.0$ (namely matter dominated) when $z \rightarrow \infty$, it is easy to find that

$$\Omega_{m0} = \left(\frac{2\beta}{2\beta + 2 - \lambda} \right)^{\frac{\lambda}{2(2 - \lambda)}}. \quad (37)$$

Therefore, Ω_{m0} is *not* an independent parameter. And then, we have

$$\Omega_m = \left[\frac{2\beta}{2\beta + (2 - \lambda) (1 + z)^{-2}} \right]^{\frac{\lambda}{2(2 - \lambda)}}, \quad (38)$$

whereas $\Omega_\Lambda = 1 - \Omega_m$. Substituting Eq. (34) into Eq. (7), we obtain the deceleration parameter

$$q \equiv -\frac{\ddot{a}}{aH^2} = \frac{1}{2} \left[1 - \frac{\lambda a^2}{2\beta + (2 - \lambda) a^2} \right]. \quad (39)$$

It is easy to see that the universe is decelerating when $(\lambda - 1) a^2 < \beta$, whereas the universe is accelerating when $(\lambda - 1) a^2 > \beta$. The transition occurs when $(\lambda - 1) a^2 = \beta$. Therefore, $\lambda > 1$ is necessary to accelerate the universe. The transition redshift is given by

$$z_t = \sqrt{\frac{\lambda - 1}{\beta}} - 1. \quad (40)$$

The effective EoS is given by (see e.g. [21, 37])

$$w_{\text{eff}} \equiv \frac{p_{\text{tot}}}{\rho_{\text{tot}}} = -1 - \frac{2}{3} \frac{\dot{H}}{H^2} = \frac{1}{3} (2q - 1) = -\frac{\lambda a^2}{3[2\beta + (2 - \lambda)a^2]}, \quad (41)$$

whereas the EoS of HDE reads

$$w_{\Lambda} = \frac{w_{\text{eff}}}{\Omega_{\Lambda}} = \frac{w_{\text{eff}}}{1 - \Omega_m}, \quad (42)$$

in which Ω_m is given in Eq. (38).

Similar to Sec. IV, we perform a joint χ^2 analysis to obtain the cosmological constraints on the model parameters λ and β , by using the combined data of the 307 Union SNIa, the shift parameter R of CMB and the distance parameter A of BAO. By minimizing the total χ^2 given in Eq. (32), we find that the best-fit parameters are $\lambda = 1.9095$ and $\beta = 0.3455$ (the corresponding $h = 0.7024$), while $\chi_{\text{min}}^2 = 311.368$. In Fig. 3, we present the 68% and 95% confidence level contours in the $\lambda - \beta$ parameter space. We also plot H , Ω_m , Ω_{Λ} , q , w_{eff} and w_{Λ} as functions of redshift z for the best-fit λ and β in Fig. 4. Obviously, the MHDE model with the parameterization given in Eq. (33) is well consistent with the observational data in fact.

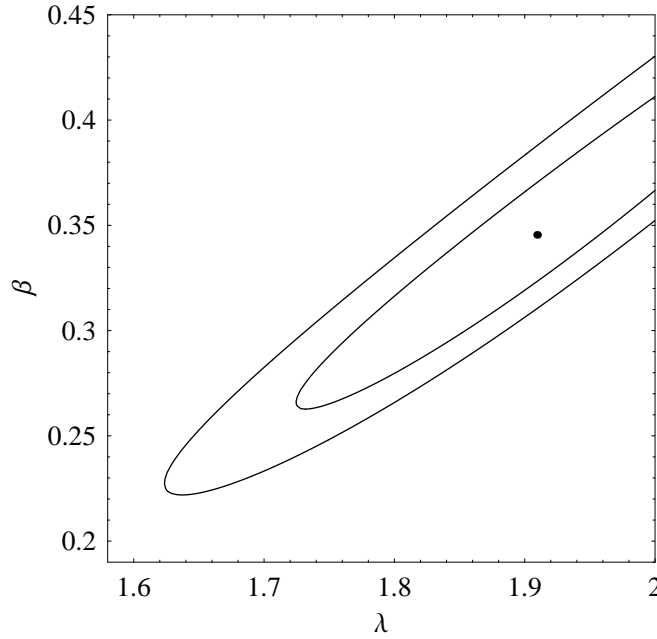


FIG. 3: The 68% and 95% confidence level contours in the $\lambda - \beta$ parameter space for the parameterization given in Eq. (33). The best-fit parameters are also indicated by a solid point.

Note that there are two parameters in the parameterization given in Eq. (33). From the viewpoint of model building, it is better to find a single-parameter model. Inspired by the above results, we can consider another parameterization

$$n^2 = 2 - \frac{3\lambda a^2}{1 + 3a^2}, \quad (43)$$

in which there is only one model parameter λ . In fact, it is just the reduced version of Eq. (33) with $\beta = 1/3$. So, all the physical quantities of the MHDE model with this parameterization can be obtained

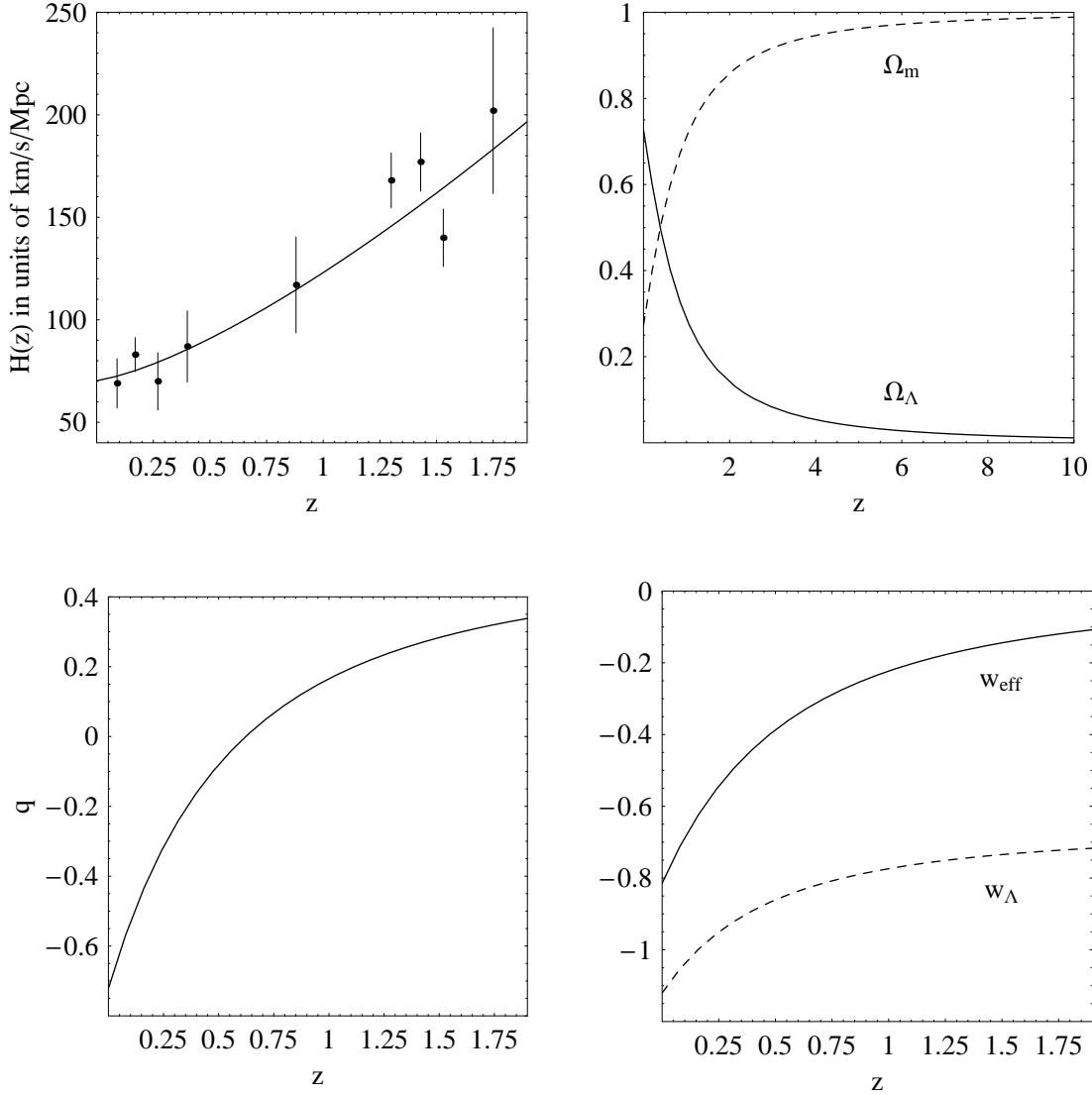


FIG. 4: The H , Ω_m , Ω_Λ , q , w_{eff} and w_Λ as functions of redshift z for the best-fit λ and β for the parameterization given in Eq. (33). These results are obtained by using the combined data of the 307 Union SNIa, the shift parameter R of CMB and the distance parameter A of BAO.

by simply setting $\beta = 1/3$ in Eqs. (34)–(42). Fitting to the combined data of the 307 Union SNIa, the shift parameter R of CMB and the distance parameter A of BAO, we find that the best-fit parameter reads $\lambda = 1.8843^{+0.0318}_{-0.0336}$ (with 1σ error) or $\lambda = 1.8843^{+0.0619}_{-0.0692}$ (with 2σ error), while $\chi^2_{\min} = 311.405$. The corresponding $h = 0.7016$. In Fig. 5, we present the corresponding likelihood $\mathcal{L} \propto e^{-\chi^2/2}$ versus λ , as well as the H , Ω_m , Ω_Λ , q , w_{eff} and w_Λ as functions of redshift z for the best-fit λ . Obviously, the MHDE model with the parameterization given in Eq. (43) is also well consistent with the observational data.

Comparing to the parameterization (12), besides the advantage of being valid for the whole $0 \leq a < \infty$, the parameterizations (33) and (43) have smaller χ^2_{\min} when we fit them to the combined data of the 307 Union SNIa, the shift parameter R of CMB and the distance parameter A of BAO.

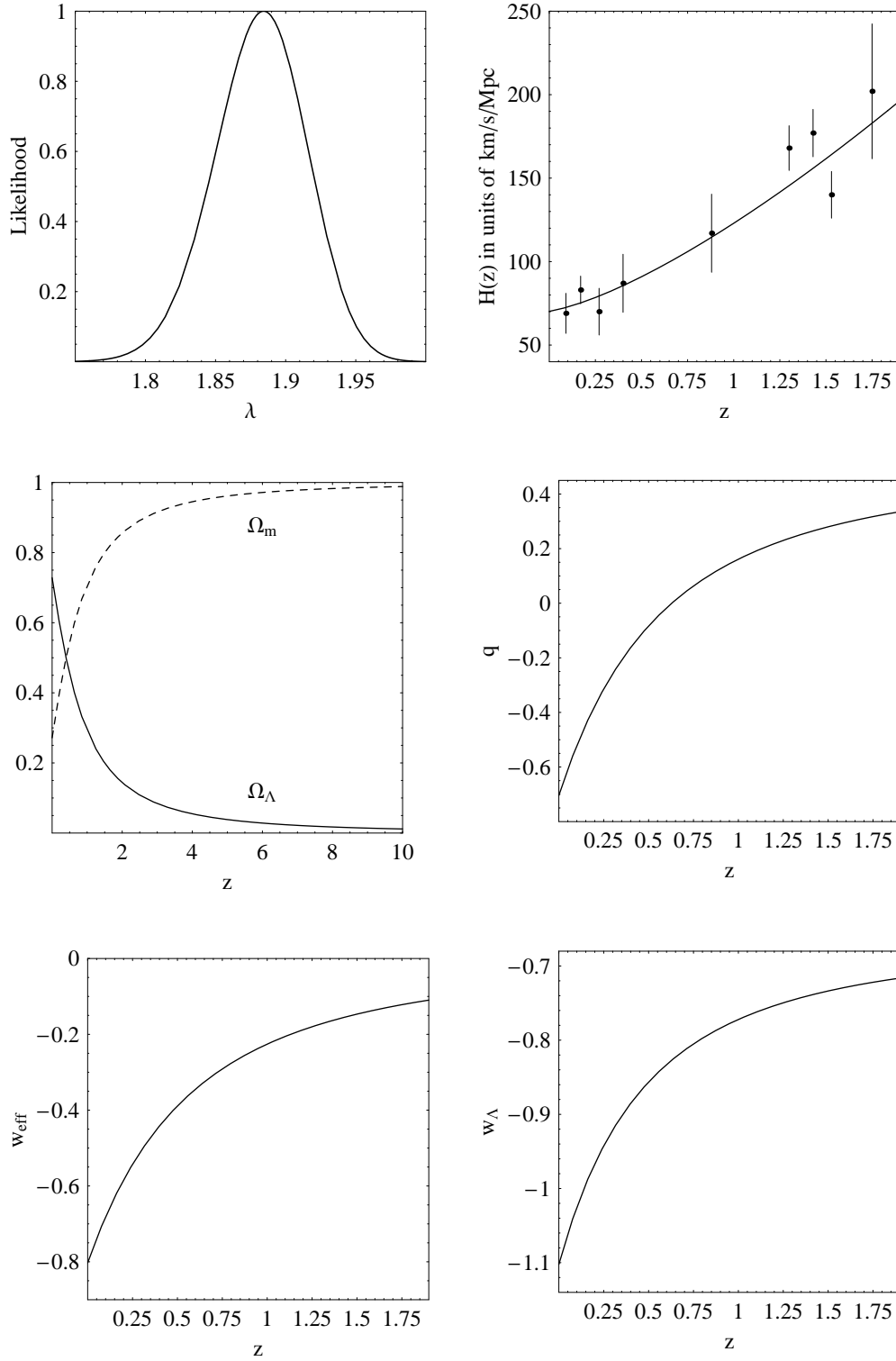


FIG. 5: The same as in Fig. 2, except for the parameterization given in Eq. (43).

VI. CONCLUSION AND DISCUSSION

In this work, motivated by the energy bound suggested by Cohen *et al.*, we propose the modified holographic dark energy (MHDE) model. Choosing the IR cut-off $L = R_{CC}$ and considering the parameterizations (12), (33) and (43), we derive all the physical quantities of the non-saturated MHDE model analytically. We find that the non-saturated MHDE models with parameterizations (12) and (43) are single-parameter models in practice. Also, we consider the cosmological constraints on the non-saturated MHDE, and find that it is well consistent with the observational data in fact. Given the simplicity and analyticity, the non-saturated MHDE model is very interesting and deserves further investigation.

After all, there are some remarks on the parameterizations of $n(a)$. Firstly, our situation is similar to the familiar parameterization for the EoS of dark energy considered extensively in the literature, such as the well-known $w(a) = w_0 + w_a(1-a)$ [20] and $w(z) = w_0 + w_1 z$ (see e.g. [27, 39, 40]). Until we can completely understand the mysterious dark energy in the far future, the method of parameterization is useful to shed some light on the nature of dark energy. Secondly, as mentioned in Sec. III, our parameterization $n^2 = 2 - \lambda a$ given in Eq. (12) is motivated by the EoS parameterization $w(a) = w_0 + w_a(1-a)$. However, as recently argued by Shafieloo *et al.* [41], the EoS parametrization $w(a) = w_0 + w_a(1-a)$ cannot be trusted in the redshift interval $0 < z < 1.4$, and it fails for $a \gg 1$. Accordingly, one might doubt our parameterization $n^2 = 2 - \lambda a$. As shown in Sec. IV, the parameterization $n^2 = 2 - \lambda a$ works well for $0 < a \leq 1$, or, $0 \leq z < \infty$. However, as mentioned in the beginning of Sec. V, it is only valid for $a < 2/\lambda$ which is required by $n^2 > 0$. Thus, we proposed other parameterizations for n^2 in Eqs. (33) and (43) which work well for the whole $0 \leq a < \infty$. Finally, one might argue that to be holographic (i.e., the energy density is proportional to the horizon area), $n(a)$ should be nearly constant. Therefore, $n(a)$ should not change so quickly. We admit that our parameterizations (12), (33) and (43) do not fulfill this point. On the other hand, they can be used as working parameterizations since they are well consistent with the observational data in fact. Nevertheless, it is still worthwhile to seek a more satisfactory parameterization for $n(a)$ which changes not so quickly and can be consistent with the observational data. We leave this to the future work. This tells us that there are still many improvements to do for the MHDE model.

ACKNOWLEDGEMENTS

We thank the anonymous referee for quite useful comments and suggestions, which help us to improve this work. We are grateful to Professors Rong-Gen Cai, Shuang-Nan Zhang, and Miao Li for helpful discussions. We also thank Minzi Feng, as well as Chang-Jun Gao, Xin Zhang, Pu-Xun Wu, Shi Qi, Shuang Wang and Xiao-Dong Li, for kind help and discussions. This work was supported by the Excellent Young Scholars Research Fund of Beijing Institute of Technology.

-
- [1] G. 't Hooft, gr-qc/9310026;
L. Susskind, J. Math. Phys. **36**, 6377 (1995) [hep-th/9409089].
 - [2] R. Bousso, Rev. Mod. Phys. **74**, 825 (2002) [hep-th/0203101].
 - [3] J. D. Bekenstein, Phys. Rev. D **7** (1973) 2333;
J. D. Bekenstein, Phys. Rev. D **9**, 3292 (1974);
J. D. Bekenstein, Phys. Rev. D **23**, 287 (1981);
J. D. Bekenstein, Phys. Rev. D **49**, 1912 (1994) [gr-qc/9307035].
 - [4] S. W. Hawking, Commun. Math. Phys. **43**, 199 (1975) [Erratum-ibid. **46**, 206 (1976)];
S. W. Hawking, Phys. Rev. D **13**, 191 (1976).
 - [5] A. G. Cohen, D. B. Kaplan and A. E. Nelson, Phys. Rev. Lett. **82**, 4971 (1999) [hep-th/9803132].

- [6] P. Horava and D. Minic, Phys. Rev. Lett. **85**, 1610 (2000) [hep-th/0001145];
S. D. Thomas, Phys. Rev. Lett. **89**, 081301 (2002).
- [7] S. D. H. Hsu, Phys. Lett. B **594**, 13 (2004) [hep-th/0403052].
- [8] W. Fischler and L. Susskind, hep-th/9806039;
R. Bousso, JHEP **9907**, 004 (1999) [hep-th/9905177].
- [9] M. Li, Phys. Lett. B **603**, 1 (2004) [hep-th/0403127].
- [10] H. Wei and S. N. Zhang, Phys. Rev. D **76**, 063003 (2007) [arXiv:0707.2129].
- [11] H. Wei, arXiv:0902.0129 [gr-qc].
- [12] X. Zhang and F. Q. Wu, Phys. Rev. D **76**, 023502 (2007) [astro-ph/0701405].
- [13] C. Gao, X. Chen and Y. G. Shen, Phys. Rev. D **79**, 043511 (2009) [arXiv:0712.1394].
- [14] C. J. Feng, arXiv:0806.0673 [hep-th];
C. J. Feng, Phys. Lett. B **670**, 231 (2008) [arXiv:0809.2502];
C. J. Feng, Phys. Lett. B **672**, 94 (2009) [arXiv:0810.2594];
C. J. Feng, arXiv:0812.2067 [hep-th].
- [15] L. Xu, W. Li and J. Lu, arXiv:0810.4730 [astro-ph];
X. Zhang, arXiv:0901.2262 [astro-ph.CO].
- [16] L. N. Granda and A. Oliveros, Phys. Lett. B **669**, 275 (2008) [arXiv:0810.3149];
L. N. Granda and A. Oliveros, Phys. Lett. B **671**, 199 (2009) [arXiv:0810.3663];
L. N. Granda and A. Oliveros, arXiv:0901.0561 [hep-th].
- [17] K. Y. Kim, H. W. Lee and Y. S. Myung, arXiv:0812.4098 [gr-qc].
- [18] R. G. Cai, B. Hu and Y. Zhang, arXiv:0812.4504 [hep-th].
- [19] B. Guberina, R. Horvat and H. Nikolic, JCAP **0701**, 012 (2007) [astro-ph/0611299].
- [20] M. Chevallier and D. Polarski, Int. J. Mod. Phys. D **10**, 213 (2001) [gr-qc/0009008];
E. V. Linder, Phys. Rev. Lett. **90**, 091301 (2003) [astro-ph/0208512].
- [21] H. Wei and S. N. Zhang, Phys. Lett. B **644**, 7 (2007) [astro-ph/0609597].
- [22] G. R. Dvali, G. Gabadadze and M. Porrati, Phys. Lett. B **485**, 208 (2000) [hep-th/0005016];
C. Deffayet, G. R. Dvali and G. Gabadadze, Phys. Rev. D **65**, 044023 (2002) [astro-ph/0105068].
- [23] H. Wei and R. G. Cai, Phys. Lett. B **663**, 1 (2008) [arXiv:0708.1894];
H. Wei and R. G. Cai, Phys. Lett. B **660**, 113 (2008) [arXiv:0708.0884].
- [24] M. Kowalski *et al.* [Supernova Cosmology Project Collaboration], arXiv:0804.4142 [astro-ph].
The numerical data of the full sample are available at <http://supernova.lbl.gov/Union>
- [25] D. Rubin *et al.* [Supernova Cosmology Project Collaboration], arXiv:0807.1108 [astro-ph].
- [26] S. Nesseris and L. Perivolaropoulos, Phys. Rev. D **72**, 123519 (2005) [astro-ph/0511040];
L. Perivolaropoulos, Phys. Rev. D **71**, 063503 (2005) [astro-ph/0412308].
- [27] E. Di Pietro and J. F. Claeskens, Mon. Not. Roy. Astron. Soc. **341**, 1299 (2003) [astro-ph/0207332].
- [28] R. Jimenez, L. Verde, T. Treu and D. Stern, Astrophys. J. **593**, 622 (2003) [astro-ph/0302560];
J. Simon, L. Verde and R. Jimenez, Phys. Rev. D **71**, 123001 (2005) [astro-ph/0412269].
- [29] L. Samushia and B. Ratra, Astrophys. J. **650**, L5 (2006) [astro-ph/0607301].
- [30] H. Wei and S. N. Zhang, Phys. Lett. B **654**, 139 (2007) [arXiv:0704.3330].
- [31] M. Tegmark *et al.* [SDSS Collaboration], Phys. Rev. D **69**, 103501 (2004) [astro-ph/0310723];
M. Tegmark *et al.* [SDSS Collaboration], Astrophys. J. **606**, 702 (2004) [astro-ph/0310725];
U. Seljak *et al.* [SDSS Collaboration], Phys. Rev. D **71**, 103515 (2005) [astro-ph/0407372];
M. Tegmark *et al.* [SDSS Collaboration], Phys. Rev. D **74**, 123507 (2006) [astro-ph/0608632].

- [32] J. R. Bond, G. Efstathiou and M. Tegmark, Mon. Not. Roy. Astron. Soc. **291**, L33 (1997) [astro-ph/9702100].
- [33] Y. Wang and P. Mukherjee, Astrophys. J. **650**, 1 (2006) [astro-ph/0604051].
- [34] E. Komatsu *et al.* [WMAP Collaboration], Astrophys. J. Suppl. **180**, 330 (2009) [arXiv:0803.0547].
- [35] D. J. Eisenstein *et al.* [SDSS Collaboration], Astrophys. J. **633**, 560 (2005) [astro-ph/0501171].
- [36] S. Nojiri and S. D. Odintsov, Gen. Rel. Grav. **38**, 1285 (2006) [hep-th/0506212];
M. Jamil, M. U. Farooq and M. A. Rashid, arXiv:0901.2482 [gr-qc];
M. A. Rashid, M. U. Farooq and M. Jamil, arXiv:0901.3724 [gr-qc];
W. Zhao, Phys. Lett. B **655**, 97 (2007) [arXiv:0706.2211].
- [37] H. Wei and R. G. Cai, Eur. Phys. J. C **59**, 99 (2009) [arXiv:0707.4052].
- [38] D. Pavon and W. Zimdahl, Phys. Lett. B **628**, 206 (2005) [gr-qc/0505020];
N. Banerjee and D. Pavon, Phys. Lett. B **647**, 477 (2007) [gr-qc/0702110];
D. Pavon, J. Phys. A **40**, 6865 (2007) [gr-qc/0610008];
W. Zimdahl and D. Pavon, Class. Quant. Grav. **24**, 5461 (2007).
- [39] M. Goliath *et al.*, Astron. Astrophys. **380**, 6 (2001) [astro-ph/0104009];
I. Maor, R. Brustein and P. J. Steinhardt, Phys. Rev. Lett. **86**, 6 (2001) [astro-ph/0007297];
I. Maor, R. Brustein, J. McMahon and P. J. Steinhardt, Phys. Rev. D **65**, 123003 (2002) [astro-ph/0112526].
- [40] A. G. Riess *et al.* [Supernova Search Team Collaboration], Astrophys. J. **607**, 665 (2004) [astro-ph/0402512].
- [41] A. Shafieloo, V. Sahni and A. A. Starobinsky, arXiv:0903.5141 [astro-ph.CO].